

Lecture 7: Incongruent Counterparts

7.1 Kant's 1768 paper

7.1.1 The Leibnizian background

Although Leibniz ultimately held that the phenomenal world, of spatially extended bodies standing in various distance relations from one another, was not fundamental, he believed that our everyday notion of space as something independent of bodies was an abstraction from *this* order. That all spatial facts are to be reducible to facts about the relative distances between material bodies is the defining thesis of the classical relationalist. Kant's 1768 paper certainly has this position in its sights. But the thinking behind Kant's paper is perhaps made more transparent when one takes into account some key features of Leibniz's *analysis situs*. In this, the fundamental notion in analyzing spatial qualities was to be the relation of *congruence*, which Leibniz understood in terms of *equality of magnitude* and *similarity of form*.

I shall here use the relations of congruence, which I shall express by the character \cong . For example, in the first figure (Figure 1), $ABC \cong DEF$ means that triangles ABC and DEF are congruent with respect to the order of their points, that they can occupy exactly the same place, and that one can be applied or placed on the other without changing anything in the two figures except their place. (Leibniz 1969, 251)

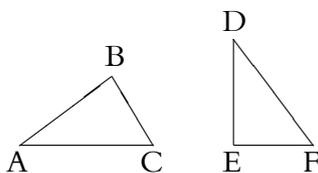


Fig. 1

Kant initially defended a Leibnizian account of space, but he became disaffected with it during the 1760s.

7.1.2 The significance of orientation

Kant aims to convince us that there is a “clear proof” that:

Absolute space, independently of the existence of all matter. . . has a reality of its own. (Kant 1992a, 366; Kant's own emphasis)

He had come to believe that specifying a system's *relative configuration* (i.e. the relative distances between the parts of the system), did not exhaust all there was to say. Reflection on such things as maps was meant to suggest the necessity of specifying *the orientation of a system*, in addition to its relative configuration:

... the most precise map of the heavens, if it did not, in addition to specifying the positions of the stars relative to each other, also specify the direction by reference to the position of the chart relative to my hands, would not enable me, no matter how precisely I had it in mind, to infer from a known direction, for example, the north, which side of the horizon I ought to expect the sun to rise. The same thing holds of geographical and, indeed, of our most ordinary knowledge of the position of places. Such knowledge would be of no use unless we could also orientate the things thus ordered, along with the entire system of their reciprocal positions, by referring them to the sides of our body. (Kant 1992a, 367–8)

So, in addition to relative distances, Kant thought that the notion of *direction* was fundamental. To fully specify a body's spatial characteristics, one needs to say how it is oriented and this is ultimately supposed to depend on how the body is related to space as whole:

... the positions of the parts of space in reference to each other presuppose the direction in which they are ordered in such relation. . . In the case of any extended thing, the position of its parts relative to each other can be adequately known by reference to the thing itself. The direction, however, in which this order of parts is orientated, refers to the space outside the thing. To be specific: it refers not to places in the space – for that would be the same thing as regarding the position of the parts to the thing in question in an external relation – but rather to universal space as a unity. . . (Kant 1992a, 365)

What we are trying to demonstrate, then, is the following claim. The ground of the complete determination of a corporeal form does not depend simply on the relation and position of its parts to each other; it also depends on the reference of that physical form to universal absolute space, as it is conceived by the geometers. (Kant 1992a, 369)

Incongruent counterparts then play a key role in showing that facts about orientation go beyond facts about relative distances:

If two figures drawn on a plane surface are equal and similar, then they will coincide with each other. But the situation is often entirely different when one is dealing with corporeal [bodies]. . . They can be exactly equal and similar, and yet still be so different in themselves that the limits of the one cannot also be the limits of the other [. . .] [T]he most common and clearest example is furnished by the limbs of the human body. . . The right hand is similar and equal to the left hand. And if one looks at one of them on its own, examining the proportion and the position of its parts to each other, and scrutinising the magnitude of the whole, then a complete description of the one must apply in all respects to the other, as well.

I shall call a body which is exactly equal and similar to another, but which cannot be enclosed in the same limits as that other, its *incongruent counterpart*. (Kant 1992a, 369–70)

7.1.3 The argument

Kant's "philosophical application" of incongruent counterparts runs as follows:

It is apparent from the ordinary example of the two hands that the shape of the one body may be perfectly similar to the shape of the other, and the magnitudes of their extensions may be exactly equal, and yet there may remain an inner difference between the two, this difference consisting in the fact, namely, that the surface which encloses the one cannot possibly enclose the other. Since the surface which limits the physical space of the one body cannot serve as a boundary to limit the other, no matter how that surface be twisted and turned, it follows that the difference must be one which rests on an inner ground. This inner ground cannot, however, depend on the difference of the manner in which the parts of the body are combined with each other. For as we have seen from our example, everything may in this respect be exactly the same. **Nonetheless, imagine that the first created thing was a human hand. That [hand] would have to be either a right hand or a left hand. The action of the creative cause in producing the one would have of necessity to be different from the action of the creative cause in producing the counterpart.**

Suppose that one were to adopt the concept entertained by many modern philosophers, especially German philosophers, according to which space simply consists in the external relation of the parts of matter which exist alongside each other. . . [Since] there is no difference in the relation of the parts of the hand to each other, and that is so whether it be a right hand or a left hand; it would therefore follow that the hand would be completely indeterminate in respect of such a property. In other words, the hand would fit equally well on either side of the human body; but that is impossible. (Kant 1992a, 370–1)

7.1.4 Immediate comments

1. How absurd is it to suppose that the lone hand would fit equally well on either side of the human body? (Cf. Remnant 1963; Dainton 2001, 227–31)
2. What, exactly, is the nature of the relation which left hands bear to space but which right hands don't? Hofer suggests that haecceitist substantialists have an answer (Hofer 2000, 241ff).

7.1.5 What the relationalist needs to do:

1. Explain the *incongruence* of counterparts;
2. Explain what it is to be of *one particular handedness* (in a way that is consistent with the denial of Kant's main premise);
3. Explain how we can use maps.

7.2 Nerlich's argument

Another related spatial property one might wish to give an account of is that shared by incongruent counterparts but not shared by objects that can only have *congruent* counterparts. Kant suggests that to be an object that can have incongruent counterparts—to be an *enantiomorph* rather than a *homomorph*—an object must not consist of “two halves which are symmetrically arranged relatively to a single intersecting plane” (Kant 1992a, 370). But as Nerlich points out (Nerlich 1994, Ch. 2), this is merely a necessary condition: the *dimensionality and orientability* of space are also crucial. Here substantial space plays an obvious role that is independent of haecceitism.

Nerlich's variant of Kant's claim is that were the first created thing a human hand, it would have to be either a homomorph or an enantiomorph. Let us return to this challenge later. . .

7.3 Intrinsic versus extrinsic

Despite talk of “inner differences” Kant seemed to regard being of a particular handedness as an *extrinsic* matter. There is clearly an alternative:

Incongruent counterparts differ intrinsically (such differences are to be exploited in explanations of their incongruence and of what it is to be of one particular handedness)

Incongruent counterparts are intrinsically identical (in which case explanations of incongruence and of the handedness must appeal to something *extrinsic* to the handed object)

Problems with the first option (see Nerlich 1994, 51–3; Frederick 1991, 8; Van Cleve 1991, 22–3; Earman 1971, 134–9):

1. higher-dimensional spaces
2. non-orientable spaces
3. epistemology

The second, parsimonious, option is thus overwhelmingly more attractive.

7.4 A relational account of handedness

7.4.1 The relational basis of incongruence

The notions of *rigid motion*, and of one body's being able to occupy the space of another, are obviously relationally acceptable. It is simply a *mathematical* fact, and a perfectly *comprehensible* one that, *when constrained to obey the algebraic relationships of Euclidean geometry*, some numbers (the possible distances between two congruent counterparts) can be continuously altered so as to vanish while others (the distances between two incongruent counterparts) cannot.

Which things in a given space lie in one direction and which things incline in the opposite direction cannot be described discursively nor reduced to characteristic marks of the understanding by any astuteness of the mind. Thus, between solid bodies which are perfectly similar and equal but incongruent, such as left and right hands (in so far as they are conceived only according to their extension), or spherical triangles from two opposite hemispheres, there is a difference, in virtue of which it is impossible that the limits of their extension should coincide – and that, in spite of the fact that, in respect of everything which may be expressed by means of characteristic marks intelligible to the mind through speech, they could be substituted for one another. It is, therefore, clear that in these cases the difference, namely the incongruity, can only be apprehended by a certain pure intuition. (Kant 1992b, 396)

7.4.2 A relational account of handedness

(Compare: Earman 1989, Ch. 7, §2; Gardner 1990, Ch. 17; Huggett 1999, 209–12; Hofer 2000, §3; and Saunders 2000, §3.3.)

1. Incongruent counterparts are intrinsically identical.
2. Their incongruence can be understood (*pace* Kant) in terms of the spatial relations that hold between them.
3. Each set of handed objects of a particular type is partitioned into two sets by the equivalence relation of congruence.
4. “Left” and “right” are ‘proper names’ for members of one or other partition: we recognise objects as being of one or another handedness by recognising its (in)congruence with similar objects that we have learned through ostension are left(/right)-handed.

7.4.3 The ‘Kantian Hypothesis’ and the ‘Ozma Problem’

Jonathan Bennett dubs the claim that the difference between left and right (*in a which is which sense*) can be explained “only by showing and not by telling” the *Kantian Hypothesis* (Bennett 1970). Suppose that we are in radio contact with some extra-galactic civilization:

Is there any way to communicate the meaning of “left” by a language transmitted in the form of pulsating signals? By the terms of the problem we may say anything we please to our listeners, ask them to perform any experiment whatever, with one proviso: *There is to be no asymmetric object or structure that we and they can observe in common.* (Gardner 1990, 167)

7.4.4 Outstanding problems for the reductive relationalist

Recall Nerlich's challenge. The 'relational' account offered has exploited various constraints on the relative distances (specifically embeddability in Euclidean of space of a fixed dimension and determinate orientability). Can the relationalist do any better? (See [Brighouse 1999](#); [Pooley 2002](#), Ch. 6; [Huggett 2003](#).)

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