Lecture 3: General Covariance and the Hole Argument

3.1 Substantivalism and relationalism

Substantivalism: space (or spacetime) is a substance

(Reductive) Relationalism: the denial of substantivalism (not to be confused with 'non-reductive relationalism' (Saunders 2002)).

> ... [In the relativistic context] Substantivalists understand the existence of spacetime in terms of the existence of its pointlike parts, and gloss spatiotemporal relations between material events in terms of the spatiotemporal relations between points at which they occur. Relationists will deny that spacetime points enjoy this robust sort of existence, and will accept spatiotemporal relations between events as primitive. (Belot & Earman 2001, 227)

> ... a modern-day substantivalist thinks that space-time is a kind of thing which can, in consistency with the laws of nature, exist independently of material things (ordinary matter, light, and so on) and which is properly described as having its own properties, over and above the properties of any material things that may occupy parts of it. (Hoefer 1996, 5)

> Some substantivalists, at least, will affirm, while all relationalists will deny, that there are distinct possible worlds in which the same geometries are instantiated, but which are nonetheless distinct in virtue of the fact that different roles are played by different spacetime points (in this world, the maximum curvature occurs at *this* point, while it occurs at *that* point in the other world). We will call substantivalists who go along with these sort of counterfactuals *straightforward* substantivalists.(Belot & Earman 2001, 228)

3.2 Leibniz's argument

1. If space is "something in itself, besides the order of bodies among themselves" then there exists a non-actual (physically) possible world W' that differs from the actual world $W_@$ solely in virtue of where everything is located in absolute space.
2. If space is something in itself then there exists a non-actual (physically) possible world W'' that differs from the actual world in virtue of the absolute velocities of all bodies being (uniformly) different from their actual velocities.
3. $W_@$, W' and W'' are indiscernible(/violate the Principle of Sufficient Reason).
4. The PII (PSR) is true.
5. If space is nothing over and above the order of bodies among themselves, there are no such non-actual worlds as W' and W''.
6. Therefore, space is nothing over and above the order of bodies.

3.2.1 Haecceitism

Haecceitism is the view that possible worlds can differ solely over which objects instantiate which properties. Two possible worlds that contain exactly the same individuals and the same patterns of property instantiation, but that have different individuals instantiating certain properties, differ purely haecceitistically. (*Cf.* Lewis 1986, 221)

Note that the difference between $W_@$ and W' is purely haecceitistic. The differences between them and W'' are not.

3.3 General relativity

3.3.1 Manifolds etc.

For an introduction to the mathematics, see one of Stewart (1991, Ch. 1), Friedman (1983, 340ff), Torretti (1983, 257ff), Wald (1984, 423–7; 437–44).

A differential manifold is a topological space, M, together with a maximal atlas of charts, A. A chart is just a coordinate system, i.e., a map $\phi : U \subset M \rightarrow \phi(U) \subset \mathbb{R}^n$. (If every chart $\phi \in A : U \rightarrow \mathbb{R}^n$ has the same n, then the manifold is n-dimensional.)

Example: the sphere

curves: $\lambda : t \in \mathbb{I} \mapsto \lambda(t) \in M$

functions/scalar fields: $f : M \rightarrow \mathbb{R}$

the tangent vector to λ at p: $\dot{\lambda}_p : f \mapsto [\frac{d}{dt}(f \circ \lambda)]_p$. The set of all tangent vectors at p forms a vectors space $T_p(M)$, i.e., for $X_p, Y_p \in T_p(M)$ one has $(X_p + Y_p), aX_p \in T_p(M)$.

vector fields: $X : M \rightarrow T(M)$

tensors and tensor fields: $T^*_p(M) :=$ the vector space of linear maps $\omega : T_p(M) \rightarrow \mathbb{R}$. Generalizing, one considers, e.g., linear maps $S : T^*_p(M) \times \ldots \times T^*_p(M) \times T_p(M) \times \ldots \times T_p(M) \rightarrow \mathbb{R}$ and corresponding fields. In particular, a non-degenerate *metric* is a symmetric map $g : T_p(M) \times T_p(M) \rightarrow \mathbb{R}$ such that if $g(X, Y) = 0$ for all Y then $X = 0$.

diffeomorphisms $d : M \rightarrow M$

3.3.2 Passive versus active transformations

Given a diffeomorphism $\psi : M \rightarrow M$, one can consider a mapping from fields to fields, ψ^*, induced by ψ. It can be given a coordinate free definition, as follows. For a scalar field, ρ, $\psi^*\rho(\psi p) := \rho(p)$. The action of ψ^* on scalar fields can then be used to define its action on tensor fields. For example, for the vector field V, we require that $\psi^* V(\psi^*\rho)|_{\psi p} = V(\rho)|_p$ for all points p and scalar fields ρ. ψ is a purely mathematical mapping. It should not be thought of as 'moving' the points of M around. All it does is *associate* with every point p of M, another point ψp. But for a given set of fields, $\{F\}$, defined on M, it can be used in the way just described to define a *different yet related* set of fields $\{\psi^* F\}$. They are different in that F and $\psi^* F$ take different 'values' at a each point p. They are related in that the 'value' of F at p is the same as the 'value' that $\psi^* F$ has at the different point ψp.

3.3.3 Symmetries

A practitioner of mathematical physics is concerned with a certain mathematical structure and an associated set $\mathfrak{M}$ of models with this structure. The sought after laws L of physics pick out a distinguished sub-class of models $\mathfrak{M}_L := \mathrm{mod}(L) \subset \mathfrak{M}$, the models satisfying the laws L. Abstractly, a symmetry operation is a map $\mathcal{S} : \mathfrak{M} \to \mathfrak{M}$. $\mathcal{S}$ is a symmetry of the laws L just in case it preserves $\mathfrak{M}_L$, i.e. for any $m \in \mathfrak{M}_L, \mathcal{S}(m) \in \mathfrak{M}_L$. (See Earman 2002)

If one can make a distinction between the fields, A_i, that represent spacetime structure and those, P_i that represent the material contents of spacetime, one can further define:

A spacetime symmetry is a mapping that leaves all of the A_i-fields invariant, i.e., it is a "diffeomorphism ψ that maps M onto M in a way that $\psi^* A_i = A_i$ for all i" (Earman 1989, 45).

A dynamical symmetry Let $\mathcal{M} = \langle M, A_1, A_2, \ldots, P_1, P_2, \ldots\rangle$ and Φ be a diffeomorphism of M onto M, then Φ is a dynamical symmetry of L just in case for any $\mathcal{M} \in \mathfrak{M}_L$, it is also the case that $\mathcal{M}_\Phi \equiv \langle M, A_1, A_2, \ldots, \Phi^* P_1, \Phi^* P_2, \ldots\rangle$ is in $\mathfrak{M}_L$.

3.3.4 General covariance

- The laws of physics retain the same form under arbitrary coordinate transformations. (This might look like a generalization of, e.g., Lorentz covariance but beware!)

- The set of laws L is generally covariant just if, if $\langle M, g, T\rangle \in \mathfrak{M}_L$ then so is $\langle M, d^* g, d^* T\rangle$ for every diffeomorphism d.

- L is generally covariant just if *diff*(M) is a gauge group of L, i.e., just if $\langle M, g, T\rangle$ and $\langle M, d^* g, d^* T\rangle$ correspond to the same physical situation for every diffeomorphism d.

3.4 The hole argument

Choose d such that it is the identity map for all of M outside a given region $H \subset M$ (the "hole") but smoothly comes to differ from the identity map inside the hole. Call such a d a *hole diffeomorphism*. Suppose our spacetime M admits foliation by global spacelike hypersurfaces. We can label these with a continuous parameter t which increases as one moves in the future direction along any timelike curve. This effectively defines a global time function $t : M \to \mathbb{R}$, such that the level surfaces of t are the spacelike hypersurfaces. Now choose d such that it is the identity for $t \leq 0$, but is non-trivial for $t > 0$. $\langle M, g, T\rangle$ and $\langle M, d^*g, d^*T\rangle$ are then identical up to $t = 0$ but diverge thereafter.

> [O]ur argument does not stem from a conviction that determinism is or ought to be true... Rather our point is this. If a metaphysics, which forces all our theories to be deterministic, is unacceptable, then equally a metaphysics, which automatically decides in favour of indeterminism, is unacceptable. Determinism may fail, but if it fails, it should fail for a reason of physics, not because of commitment to substantival properties which can be eradicated without affecting the empirical consequences of the theory. (Earman & Norton 1987, 524)

3.4.1 Possible responses

We can distinguish three basic positions that one might adopt in regard to $\mathcal{M}_1$ and $\mathcal{M}_2$.

Haec $\mathcal{M}_1$ and $\mathcal{M}_2$ represent different physical situations, i.e., they represent different possible worlds.

LE $\mathcal{M}_1$ and $\mathcal{M}_2$ represent the same possible world.

One If $\mathcal{M}_1$ is taken to represent a possible world, then $\mathcal{M}_2$ does not represent a possible state of affairs at all; i.e., it might represent an 'impossible world.'

As an argument against substantivalism, the hole argument presupposes that the substantivalist is committed to (Haec).

Some advocate relationalism in response (e.g. Stachel 1993, Rovelli 1997). Maudlin (1989) and Butterfield (1989*b*) argued that the substantivalist should opt for (One). A more popular position is to argue that substantivalists can adopt (LE) (see Mundy 1992, Brighouse 1994, Hoefer 1996). Belot & Earman (2000) are critical of this "sophisticated" substantivalism.

3.4.2 Determinism and models

Dm2 A theory with models $\langle M, O_i\rangle$ is **S**-deterministic, where **S** is a kind of region that occurs in manifolds of the kind occurring in the models iff: given any two models $\langle M, O_i\rangle$ and $\langle M', O'_i\rangle$ containing regions S, S' of kind **S** respectively, and any diffeomorphism α from S onto S':

if $\alpha^*(O_i) = O'_i$ on $\alpha(S) = S'$, then:

there is an isomorphism β from M onto M' that sends S to S', *i.e.* $\beta^*(O_i) = O'_i$ throughout M' and $\beta(S) = S'$. (Butterfield 1989*a*, 9)

But this seems to classify as deterministic theories that are intuitively indeterministic (see Rynasiewicz 1994, Belot 1995, Melia 1999).

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